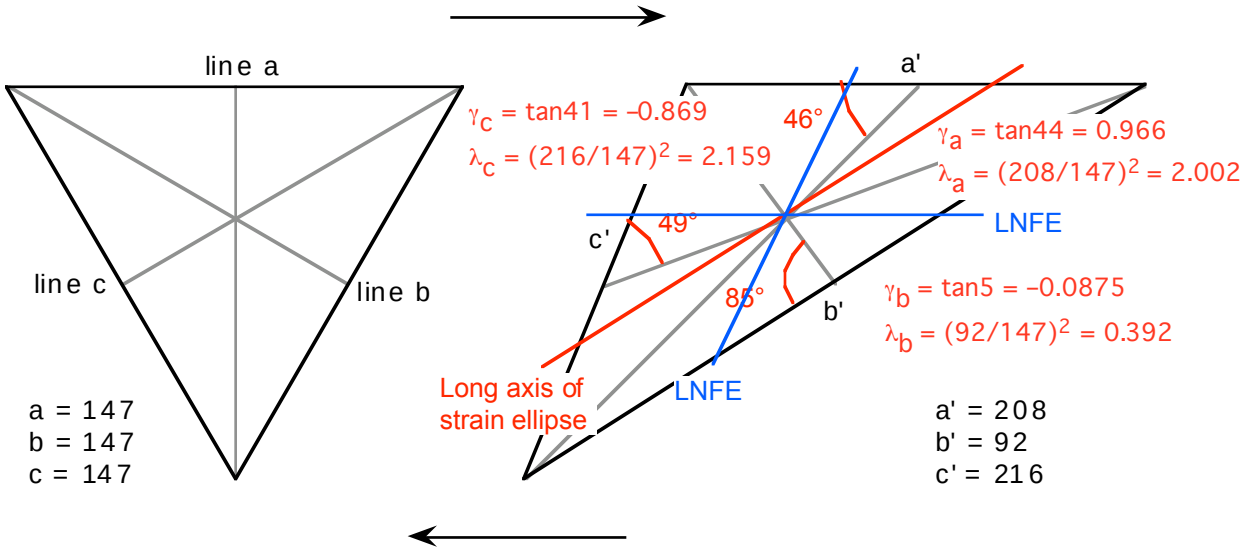
Ave = 98/150, s.d. = 21

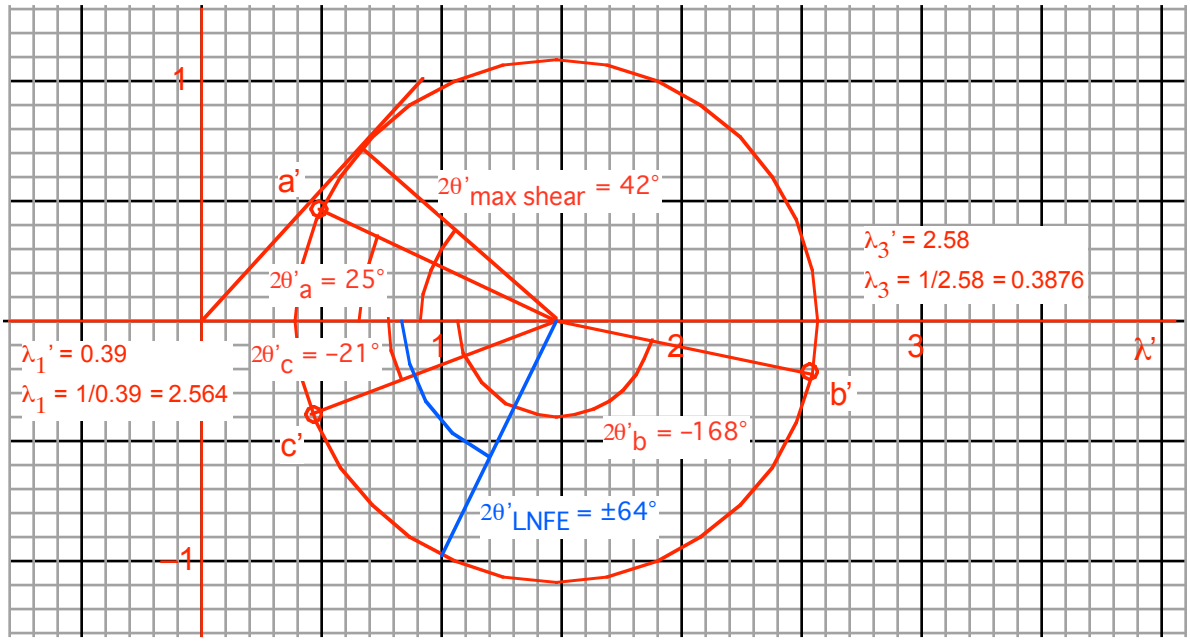
EAS 326-03 MIDTERM EXAM

This exam is closed book and closed notes. It is worth 150 points; the value of each question is shown at the end of each question. At the end of the exam, you will find two pages of potentially useful equations.

1. The following figure shows a triangle in its initial state (left) and its final state after it has been deformed by simple shear parallel to line a. The lengths of the lines before and after are given. Use the figure to answer parts 1 - 3.



a. Construct the Mohr's Circle for finite strain which describes this deformation. Clearly label the axes as well as the position of each line on the Mohr's Circle. (Hint: the perpendicular bisector of the chord of a circle goes through the center of the circle) [20 points]



b. Determine the orientations of the principal axes of strain relative to line a. [10 points]

Line	Shear strain, γ	Quadratic elongation, λ	$\gamma' = \frac{\gamma}{\lambda}$	$\lambda' = \frac{1}{\lambda}$
a	0.966	2.002	0.482	0.499
b	−0.0875	0.392	−0.223	2.553
c	-0.869	2.159	−0.403	0.463

As you can see from the Mohr's Circle for strain, the $2\theta'$ angle for line a is 25° so the long axis of the strain ellipse will be oriented 12.5° from a.

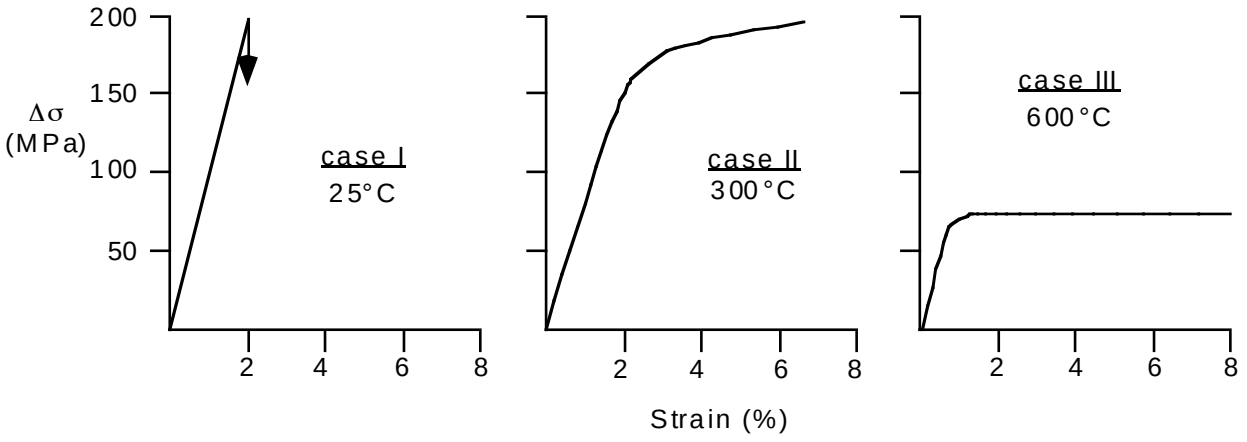
c. Determine the orientations of the two lines of no finite extension (LNFE). Show these on the Mohr's circle and on the diagram of the triangle. [10 points]

This is a simple shear deformation so the shear planes, and one of the two lines of no finite elongation (LNFE) is horizontal. Both lines must be oriented at $2\theta' = 64°$ so the two LNFE are 32° on either side of the long axis of the strain ellipse.

d. Determine the orientation of the line of maximum angular shear. [10 points]

The line of maximum angular shear, ψ, and shear strain, γ, can be found on the Mohr's Circle by drawing a line from the origin which is tangent to the circle, as shown above. This line is oriented at $2\theta' = 42°$ (or 21° from the principal extensional axis) and has an angular shear, $\psi = 48°$.

2. Below are three hypothetical stress strain curves. Assume that the deformation in case I is by Coulomb failure and that the deformation beyond yield in cases II and III is governed by crystal plastic mechanisms. In all cases, the confining pressure was 1 kbar.



ANSWER KEY

a. Describe the deformation mechanism(s) for each case. Include in your account a description of what happens at the atomic or molecular level. [30 points]

case I

In case I, the rock begins deforming elastically. Bonds between oppositely charged ions are stretched or shorten, but none are broken. If the stress is removed before it reaches 200 Mpa, the rock sample will return to its initial undeformed state, i.e., the deformation is non-permanent.

At 200 MPa, the rock fractures. It would be a relatively clean break across all the bonds in the material at once. The rock on either side of the fracture is pretty much undeformed.

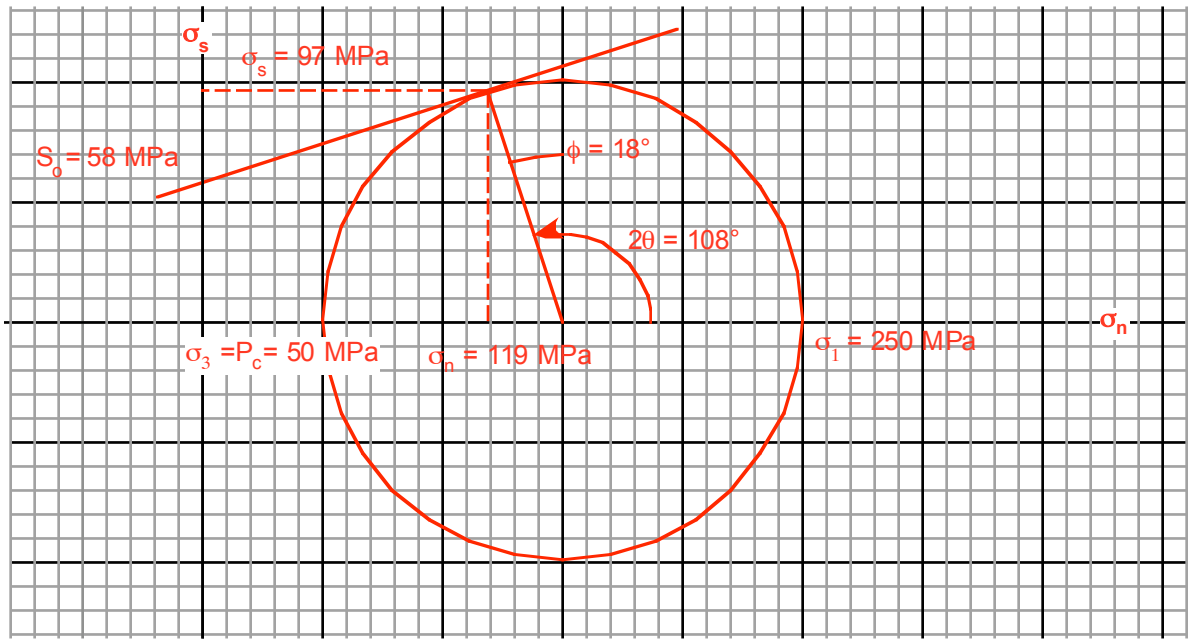
case II

In case II, elastic deformation occurs as before in case I. When the yield stress is reached at about 160 MPa, the rock begins to deform permanently, but without rupturing. Because the rock is at 300°C the deformation mechanism that is responsible for the permanent solid-state flow is probably dislocation glide. Once the dislocations begin to move, they are immediately impeded by impurity atoms, the interference of their self stress field with that of other dislocations, and by jogs produced by intersecting dislocations in different glide planes and systems. Thus it requires more stress for the dislocations to keep moving and the deformation is said to be strain (or work) hardening.

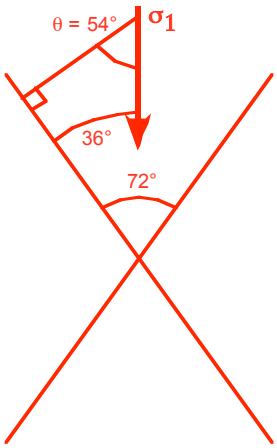
case III

Case III also starts out with some elastic deformation but because the temperatures are significantly higher the yield stress (about 75 MPa in this example) is substantially lower. When plastic deformation does start, it is **not** accompanied by strain hardening. Because the temperatures are higher, diffusion allows dislocations to climb over and around obstacles so they can keep moving freely. This is dislocation glide and climb.

b. The experiment in case I was done at a confining pressure of 50 Mpa and conjugate fractures were observed to form with an angle of 72° with respect to each other. Calculate the coefficient of internal friction (μ), the normal and shear stresses on the fracture plane at the instant of failure (σ_n and σ_s, respectively), and the cohesion (S_o). Plot the Mohr's circle for stress and the Coulomb failure envelop on the graph paper provided below. [20 points]



The differential stress at failure is $\Delta\sigma = (\sigma_1 - \sigma_3) = 200$ Mpa. The confining pressure, $P_c = \sigma_3 = 50$ Mpa, so $\sigma_1 = 250$ Mpa. As shown in the diagram at the right, σ_1 bisects the acute angle between the conjugate fractures, so $\theta = 54°$. We know that the angle of internal friction, $\phi = 2\theta - 90° = 108 - 90 = 18°$. The coefficient of internal friction, $\mu = \tan\phi = \tan 18 = 0.325$. The normal and shear stress at failure, as well as the cohesion can be read directly off of the Mohr's Circle for stress above. You can also calculate them from the equations for Mohr's Circle and for the Coulomb failure envelope.



ANSWER KEY

3. Define the following terms and describe their importance in a geological context [10 pts, each]:

a. Spherical stress

This is a special state of stress encountered in fluids which cannot support shear stress so every plane is perpendicular to a principal stress, and all principal stresses are equal. The mean stress is equal to any of the three principal stresses which is just the pressure in the fluid.

b. First Fresnel zone

The first Fresnel zone defines the minimum horizontal dimension resolvable with acoustic waves used in standard seismic reflection surveys. At horizontal dimensions less than the first Fresnel zone reflecting interfaces appear as point sources and produce diffractions rather than reflections. Diffractions can be useful to the structural geologist interpreting seismic reflection profiles because they can be use to identify truncations at fault planes that are too steep to be imaged directly with the reflection technique.

c. Plane strain

Where there is no change in shape or dimensions in the third dimension the strain is said to be plane strain. The intermediate principal axis of the finite strain ellipse has a stretch = 1 and an extension = 0

d. Stylolite

An irregular, "sawtooth" surface across which soluble rock material (usually the minerals calcite or quartz) has been dissolved, leaving an insoluble residue (usually clays). Stylolites are produced by pressure solution: preferential dissolution in a directed stress field. Material is dissolved from points of high concentration and either redeposited at points of low stress concentration or if enough water is available flushed out of the system. The former produces a volume constant deformation whereas the latter results in an overall volume loss on planes approximately perpendicular to the maximum principal stress.

e. Cauchy's law

Cauchy's law: $p_i = \sigma_{ij} l_j$, shows that stress, σ_{ij} , is a second order tensor that relates two vectors: p_i , the traction (or stress vector) on the plane and l_j , the pole to the plane (in direction cosines).

Potentially Useful Equations

Note that not all of these equations are needed for the exam and that some of them have not, or will not, be covered in class.

$$\sigma_{\Delta T} = \frac{\alpha E \Delta T}{1-\upsilon}$$

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

$$V_i = \frac{k_{ij}}{\eta}\left(\frac{dP_P}{dx_j}\right)$$

$$\sigma_n^* = \left(\frac{\sigma_1^* + \sigma_3^*}{2}\right) + \left(\frac{\sigma_1^* - \sigma_3^*}{2}\right)\cos 2\theta$$

$$\sigma_s = \left(\frac{\sigma_1^* - \sigma_3^*}{2}\right)\sin 2\theta$$

$$\lambda' = \left(\frac{\lambda_3' + \lambda_1'}{2}\right) - \left(\frac{\lambda_3' - \lambda_1'}{2}\right)\cos 2\theta'$$

$$\gamma' = \left(\frac{\lambda_3' - \lambda_1'}{2}\right)\sin 2\theta'$$

$$\tan\theta' = \tan\theta\frac{\sqrt{\lambda_3}}{\sqrt{\lambda_1}} = \tan\theta\frac{S_3}{S_1}$$

$$\sigma_s = S_o + \sigma_n^*\mu$$

$$\dot{\varepsilon} = C_o(\sigma_1 - \sigma_3)^n \exp\left(\frac{-Q}{RT}\right)$$

$$\dot{\varepsilon} = C_o(T)\frac{D(\sigma_1 - \sigma_3)}{d^n}$$

$$U = -\frac{C_1}{r} + \frac{C_2}{r^{12}}$$

$$P_{lith} = \int_0^z \rho g dz$$

$$\phi = \frac{V_f}{V_f + V_s}$$

$$\phi = \phi_o \exp(-az)$$

$$\Delta_v = \frac{V_{final} - V_{initial}}{V_{initial}}$$

$$e = \frac{l_f - l_i}{l_i}$$

$$e = \frac{\sin(\phi + \theta)}{\sin\phi} - 1$$

$$S = \frac{l_f}{l_i} = \sqrt{\lambda}$$

$$\lambda = S^2$$

$$\lambda' = \frac{1}{\lambda}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos^2\theta = \left(\frac{1+\cos 2\theta}{2}\right)$$

$$\sin^2\theta = \left(\frac{1-\cos 2\theta}{2}\right)$$

$$U_i = U_{o_i} + E_{ij}dx_j$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} U_{o_1} \\ U_{o_2} \\ U_{o_3} \end{bmatrix} + \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix}\begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix}$$

$$\sigma_m^* = \frac{(\sigma_1 + \sigma_2 + \sigma_3 - 3P_f)}{3}$$

$$\sigma_1^* = C_o + K\sigma_3^*$$

$$K = \frac{1+\sin\phi}{1-\sin\phi} \;;\quad C_o = 2S_o\sqrt{K}$$

$$\sigma_\tau = 0.85\sigma_n^*$$

$$\sigma_\tau = 50\ \text{MPa} + 0.6\sigma_n^*$$

$$(\alpha+\beta) = \frac{(1-\lambda_f)\mu_f + \beta}{(1-\lambda)k+1}$$

R = 8.3144 x 10^{-3} kJ/mol °K
= 1.9872 x 10^{-3} kcal/mol °K

°K = °C + 273.16°

1 MPa = 106 kg/m s^2 = 10 bars

g = 9.8 m/s^2 = 980 cm/s^2

$$\cos\alpha = \cos(trend)\cos(plunge)$$
$$\cos\beta = \sin(trend)\cos(plunge)$$
$$\cos\gamma = \sin(plunge)$$

$$\cos\alpha = \sin(strike)\sin(dip)$$
$$\cos\beta = -\cos(strike)\sin(dip)$$
$$\cos\gamma = \cos(dip)$$

$$\tan 2\theta' = \frac{2}{\gamma}$$

$$p_i = \sigma_{ij} l_j$$

$$p_1 = \sigma_{11}l_1 + \sigma_{12}l_2 + \sigma_{13}l_3$$
$$p_2 = \sigma_{21}l_1 + \sigma_{22}l_2 + \sigma_{23}l_3$$
$$p_3 = \sigma_{31}l_1 + \sigma_{32}l_2 + \sigma_{33}l_3$$

$$L_d = 2\pi T_3\sqrt{\frac{E}{6E_o}}$$

$$L_d = 2\pi T_3\sqrt{\frac{\eta(S-1)}{6\eta_o(2S^2)}}$$

$$C \equiv \frac{1}{r}$$

$$C_G = C_{\max}C_{\min}$$

$$\beta = \theta - \phi + (180° - 2\gamma)$$

$$\phi = \tan^{-1}\left\{\frac{-\sin(\gamma-\theta)[\sin(2\gamma-\theta)-\sin\theta]}{\cos(\gamma-\theta)[\sin(2\gamma-\theta)-\sin\theta]-\sin\gamma}\right\}$$

$$\phi = \theta = \tan^{-1}\left\{\frac{\sin(2\gamma)}{2\cos^2(\gamma)-1}\right\}$$

$$\gamma = \tan\psi = 2\tan\left(\frac{\delta}{2}\right)$$

$$\gamma = \tan\psi \cong 0.0175(\delta)$$

$$\bar{s} = 2\bar{h}\tan\left(\frac{\delta}{2}\right)$$

$$\bar{s} \cong 0.0175\bar{h}(\delta)$$

$$\Sigma\,\Delta M = 0 = \Delta M_w + \Delta M_s + \Delta M_c + \Delta M_m + \Delta M_a$$

$$0 = \Delta(\rho_w h_w) + \Delta(\rho_s h_s) + \Delta(\rho_c h_c) + \Delta(\rho_m h_m) + \Delta(\rho_a h_a)$$

$$\Delta E = \Delta h_w + \Delta h_s + \Delta h_c + \Delta h_m + \Delta h_a$$